Fishbone simulation using M3D-C1-K with thermal ion kinetic effects

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Energetic Particle Seminar

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Outline

1. New kinetic-MHD coupling scheme including thermal ions
2. Kinetic-MHD simulation of fishbone modes with thermal ions
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M3D-C1-K: Kinetic module in M3D-C1 for energetic particle simulation

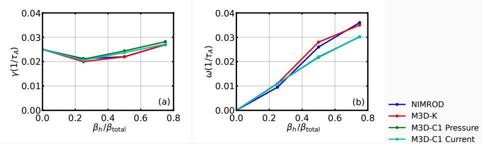
- M3D-C1-K is a kinetic-MHD code based on the MHD code M3D-C1 developed at PPPL, using gyrokinetic model and particle-in-cell (PIC) for particles, similar to codes like M3D-K, MEGA and HYM.
  - The module includes particle pushing, weight calculation (δf method), and coupling with MHD equations.
  - The code has been optimized for GPUs
  - We use pressure coupling to couple with MHD equations. It is proved to be equivalent to current coupling implemented in MEGA except for non-adiabatic particle response.
- It can be used to study the interaction between EPs and MHD activities (Alfvén waves, fishbone modes etc).

Pressure coupling vs. current coupling

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t}\right)+\rho(\mathbf{V}\cdot\nabla\mathbf{V}) = \mathbf{J}\times\mathbf{B}-\nabla p-\nabla\cdot\mathbf{P}_h$$

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t}\right)+(\mathbf{V}\cdot\nabla\mathbf{V}) = (\mathbf{J}-\mathbf{J}_h)\times\mathbf{B}-\nabla p$$

Growth rate and real frequency of fishbone from linear simulation



C. Liu, S.C. Jardin, H. Qin, J. Xiao, N.M. Ferraro, and J. Breslau, Computer Physics Communications 275, 108313 (2022).

Extending M3D-C1-K to thermal ion kinetic simulation

- In the classical kinetic-MHD simulation framework for EP study, only the kinetic effects of fast ions are taken into account.
  - The bulk plasma dominated by the thermal ions are simulated with MHD equations, while the fast ions are simulated as kinetic particles
  - The classical coupling scheme is based on the assumption that EPs only takes a small portion of ion density.
- Kinetic effect of thermal ions becomes important for future fusion devices
  - For $T_i > 10$keV, the thermal ions themselves can drive Alfven eigenmodes which can lead to minor disruptions (Du et al., PRL 2021)
  - For lower frequency EP driven modes (fishbones, BAAE), the Landau damping effect from thermal ions can suppress the mode growth.

Classical kinetic-MHD model in M3D-C1-K based on pressure coupling

MHD equation: $$\rho\left[\frac{\partial\mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right]=\mathbf{J}\times\mathbf{B}-\nabla_\perp p-\nabla_\perp\cdot\left[P_{f\parallel}\mathbf{bb}+P_{f\perp}\left(\mathbf{I}-\mathbf{bb}\right)\right]+\nu\nabla^2\mathbf{v}$$

Particle motion: $$\frac{d\mathbf{X}}{dt}=\frac{1}{B^\star}\left[V_\parallel\mathbf{B}^\star-\mathbf{b}\times\left(\mathbf{E}-\frac{\mu}{q}\nabla B\right)\right]$$

$$m\frac{dV_\parallel}{dt}=\frac{1}{B^\star}\mathbf{B}^\star\cdot\left(q\mathbf{E}-\mu\nabla B\right)$$

Weight equation: $$\frac{dw}{dt}=-(1-w)\frac{1}{f_0}\frac{df_0}{dt}$$

Particle pressure: $$\delta P_{\parallel,f}(\mathbf{x})=\sum_k mV_{\parallel,k}^2\left(w_k+\frac{\delta B_\parallel}{B_0^\star}\right)S\left(\mathbf{x}-\mathbf{x}_k\right)$$

$$\delta P_{\perp,f}(\mathbf{x})=\sum_k \mu_k B_0\left(w_k+\frac{\delta B_\parallel}{B_0^\star}+\frac{\delta B_\parallel}{B_0}\right)S\left(\mathbf{x}-\mathbf{x}_k\right)$$

Classical kinetic-MHD model in M3D-C1-K based on pressure coupling

MHD equation: $$\rho\left[\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right]+\frac{d\mathbf{K}_f}{dt}=\mathbf{J}\times\mathbf{B}-\nabla_\perp p-\nabla_\perp\cdot\left[P_{f\parallel}\mathbf{bb}+P_{f\perp}\left(\mathbf{I}-\mathbf{bb}\right)\right]+\nu\nabla^2\mathbf{v}$$

Particle motion: $$\frac{d\mathbf{X}}{dt}=\frac{1}{B^\star}\left[V_\parallel\mathbf{B}^\star-\mathbf{b}\times\left(\mathbf{E}-\frac{\mu}{q}\nabla B\right)\right]$$

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Coupling scheme with kinetic thermal ions inspired by Sato 2019&2020

$$\rho\left[\frac{\partial \mathbf{v}_\perp}{\partial t} + \left(\mathbf{v}_\perp + \mathbf{v}_\parallel \mathbf{b}\right)\cdot\nabla\mathbf{v}_\perp\right] = \mathbf{J}\times\mathbf{B} - \nabla_\perp p_e - \nabla_\perp\cdot\left[P_{i\parallel}\mathbf{bb} + P_{i\perp}\left(\mathbf{I} - \mathbf{bb}\right)\right]$$
$$- \nabla_\perp\cdot\left[P_{f\parallel}\mathbf{bb} + P_{f\perp}\left(\mathbf{I} - \mathbf{bb}\right)\right] + \nu\nabla^2\mathbf{v}_\perp,$$

$$\delta n_{i,f}(\mathbf{x}) = \sum_{k_{i,f}}\left(w_{k_{i,f}} + \frac{\delta B_\parallel}{B_0^\star}\right) S\left(\mathbf{x} - \mathbf{x}_{k_{i,f}}\right),$$

$$\delta n_e = Z_i \delta n_i + Z_f \delta n_f,$$

$$\delta v_\parallel(\mathbf{x}) = \frac{1}{n_{e0} + \delta n_e}\left[\sum_{k_i} Z_i V_{\parallel,k_i}\left(w_{k_i} + \frac{\delta B_\parallel}{B_0^\star}\right) S\left(\mathbf{x} - \mathbf{x}_k\right) - Z_i n_{i0} v_{\parallel i,0}\right.$$
$$\left. + \sum_{k_f} Z_f V_{\parallel,k_f}\left(w_{k_f} + \frac{\delta B_\parallel}{B_0^\star}\right) S\left(\mathbf{x} - \mathbf{x}_k\right) - Z_f n_{f0} v_{\parallel f,0}\right],$$

M. Sato and Y. Todo, Nucl. Fusion 59, 094003 (2019).
M. Sato and Y. Todo, Journal of Plasma Physics 86, (2020).

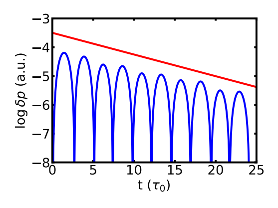
Key points of the new coupling scheme

- In addition to pressure, this scheme also uses the ion density and parallel velocity from kinetic species to substitute the corresponding MHD terms.
  - The MHD momentum equation only calculates the evolution of $\mathbf{v}_\perp$
- This scheme enforces quasineutrality condition and avoid nonphysical modes and numerical issues.
- By including the ∇p_e term in $\mathbf{E}_\parallel$, the ion motion (or the MHD $v_\parallel$) can be affected by electron pressure, which is essential for simulating ion acoustic waves, Landau damping, and kinetic Alfven waves.

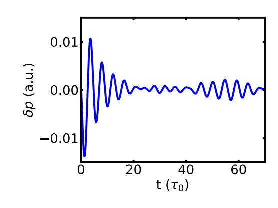
Simulation of Landau damping of ion acoustic wave

- To test the new coupling scheme, we did 1D nonlinear simulation of IAW with thermal ions only, for fixed value of $k_{\parallel}$ and different values of T_i/T_e.
- The obtained IAW frequency and damping rate are consistent with the results calculated from plasma dispersion relation.
    - The damping is very strong when T_i is close to T_e.
- For strongly damped case, the mode shows echos after damping even in linear simulation.

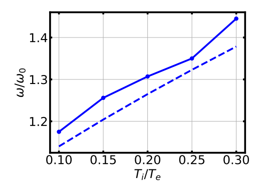
Damping of IAW from M3D-C1 simulation

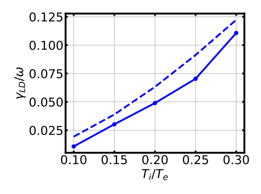


Echos of oscillation after Landau damping



IAW frequencies and damping rates as functions of T_i/T_e, comparing with analytical results (dashed)



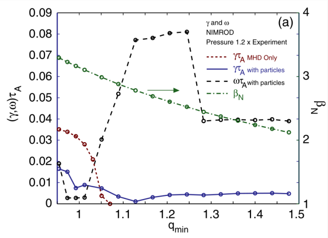


Outline

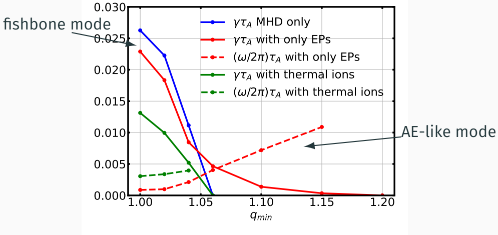
Linear simulation of fishbone mode in DIII-D #125476

- The new version of M3D-C1-K was used to simulate (1,1) kink/fishbone mode in DIII-D #125476 using an isotropic slowing-down EP distribution (16% of total pressure).
- The simulation results with only fast ions agrees with NIMROD results in Brennan 2012, including both the low-frequency fishbone mode and high-frequency AE mode.
- After including the thermal ions, both the fishbone modes and the AE modes are significantly damped.
  - Two explanations for the additional damping:
    1. precession motion of trapped thermal ions reduce the resonant response to the MHD mode
    2. Landau damping.

Growth rates and frequencies from NIMROD simulation



Growth rates and frequencies from M3D-C1 simulation



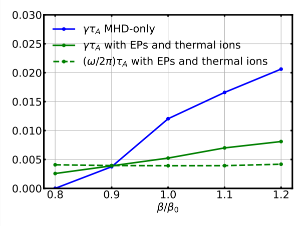
D.P. Brennan, C.C. Kim, and R.J.L. Haye, Nucl. Fusion 52, 033004 (2012).

Growth rate dependence on β becomes smoother

We did β scan of linear simulation with fixed q profile

- For MHD-only simulation, growth rates depends sensitively on β, indicating the mode is pressure-driven.
- After adding the kinetic effect, the dependence on β becomes smoother
    - For small-β case the growth rate is larger than MHD-only result.

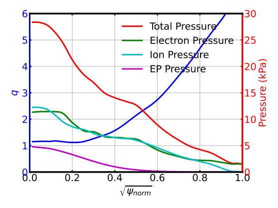
Growth rates from MHD-only and kinetic-MHD as functions of β for $q_{min} = 1.04$



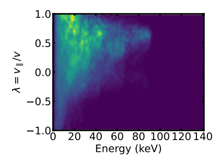
Simulation setup for kink/fishbone mode in NSTX #134020

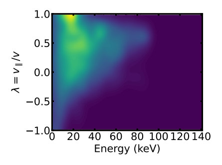
- We use M3D-C1-K to simulate (1,1) kink/fishbone mode in NSTX shot 134020 with an anisotropic EP distribution.
- The equilibrium was obtained from an EFIT output, with a modified pressure including contribution from fast ions.
  - Equilibriums were reconstructed when doing q scan to keep β fixed.
- We used a realistic 3D EP distribution from NUBEAM calculation for particle initialization and δf calculation.

Pressure and q profiles of NSTX 134020 700ms



EP distribution on magnetic axis from NUBEAM (left) and smoothed one used in M3D-C1 simulation (right)

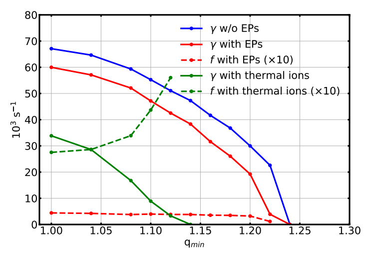




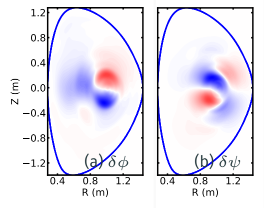
Linear simulation of fishbone growth rate/frequency

- Unlike the DIII-D simulation, the fishbone simulation with only fast ions give very small mode frequency (<1kHz), and no AE-like modes.
    - It may be caused by the lack of trapped resonant fast ions
- After including thermal ion kinetic effects, mode frequency increases due to contributions of trapped thermal ions. The growth rate decreased due to Landau damping.

Growth rates (solid) and frequencies (dashed) of fishbone as functions of q_{min}



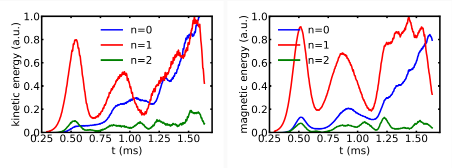
Mode structure of $n = 1$ fishbone mode



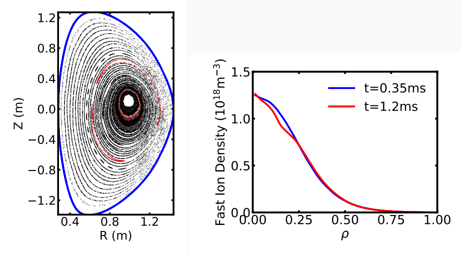
Nonlinear simulation of fishbone mode in NSTX

- In the nonlinear simulation, the $n = 1$ fishbone mode will be substituted by the $n = 0$ mode after saturation.
- For $q_{min} = 1.08$, the fishbone mode can produce stochastic fields inside $q = q_{min}$ and (2, 1) magnetic islands. Fast ion density drops 10% in the core region.
- The nonlinear saturation depends sensitively on q_{min}. For $q_{min} = 1.1$ there is almost no EP transport.

Evolution of kinetic and magnetic energy in nonlinear simulation



Poincare plot of magnetic field after fishbone excitation and transport of fast ion density

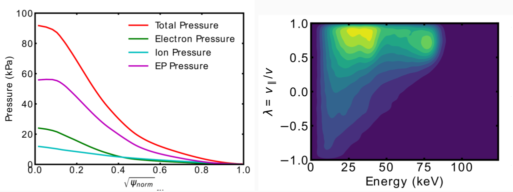


Nonlinear simulation of DIII-D #178631 fishbone modes

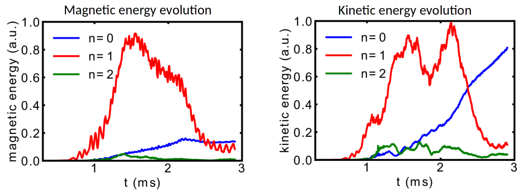
We contribute to the FY22 FES Theory Performance Target focusing on fishbone simulation in DIII-D #178631.

- This discharge has very strong NBI, with beam ions takes 31% of core ion density and 61% of core pressure.
- The $n = 1$ mode is stable for ideal MHD simulation. After adding the kinetic effects of fast and thermal ions, the $n = 1$ fishbone becomes unstable for q_{min}=1.08, with frequency $f = 24$kHz.
- Nonlinear simulation shows the mode can experience staircase-like growth before the saturation. This behavior is related to the interaction with the $n = 0$ mode.

Pressure profile and EP phase space distribution for DIII-D 178631 simulation



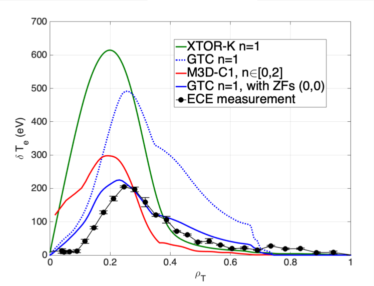
Nonlinear evolution of magnetic and kinetic energy



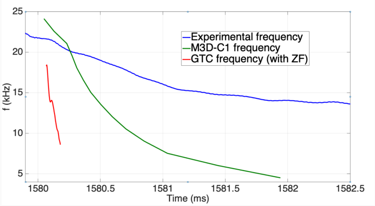
Comparison with GTC simulation results and experiments

- GTC is a gyrokinetic PIC code which has been extended to do electromagnetic simulation with finite $\delta B_{\|}$.
- Despite the difference of mode growing behavior, the two codes give close results on the saturated δT_e, in which GTC has better agreement with experiments.
- Both codes capture the frequency down-chirping of fishbone modes after saturation. The chirping rates are faster than experimental results.

Saturated δT_e from nonlinear simulations



Frequency chirping of fishbone mode



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Summary

- We have extended the kinetic-MHD model in M3D-C1 to include thermal ion kinetic effects including the parallel velocity synchronization with kinetic particles and quasi-neutrality condition. The new model can include thermal ion Landau damping self-consistently.
- It is found that thermal ions can increase the fishbone frequency and give additional damping to fishbone modes due to ion precession motion and Landau damping.

Future work:

- Study high-n pressure driven mode and impact of kinetic effects from thermal ions and fast ions.
- Investigate the effect the plasma toroidal rotation on fishbone mode excitation and saturation (Wang NF 2016)
- Add a particle source and finite diffusion term to address the discrepancy of down chirping rate with experimental results

Summary (cont'd)

- The details of kinetic-MHD simulation of fishbones with thermal ions can be found in the paper: https://arxiv.org/abs/2206.03648 (will be published in Journal of Plasma Physics soon).